Isotropic Force Control for Haptic Interfaces*

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Abstract

This paper describes a multivariable modeling and control strategy that increases the bandwidth of isotropic force transmission in multi-degree of freedom haptic interfaces. The controller structure leads to a straightforward model identification procedure and yields a simple control law that can be easily implemented. Experimental results show that the multivariable closed-loop control significantly improves performance on a 5 degree-of-freedom in-parallel haptic interface.

1 Introduction

Haptic interfaces provide forces on a user’s hand in response to hand motion. These forces can be actual forces transmitted from a remote location, as in teleoperation applications, or forces produced by a model in a computer, as in virtual reality applications.

An ideal haptic interface should have a large range of motion in multiple degrees of freedom, and accurate transmission of desired forces from the remote or virtual environment to the user. Forces due to interface dynamics, such as friction and mass, mask the transmitted forces. Quantitative requirements for such a transparent haptic interface depend on human haptic sensitivity, since forces below sensory thresholds do not contribute to loss of transparency. Sensitivity to these dynamic forces is a complex issue, and little psychophysical data is available [7]. The existing data does suggest that a truly transparent interface would be extremely difficult to achieve. The work of [1, 13] shows that extremely small vibrations (motions, not forces) can be detected, with sensitivity increasing to a maximum at about 300 Hz. This suggests that a transparent interface must produce accurate forces on the fingers up to a bandwidth exceeding 300 Hz, with large range of motion, and in multiple degrees of freedom. Such performance from a mechanical system is daunting, even in one degree of freedom [3], or with a small range of motion [4]. However, there are several factors mitigating the high bandwidth requirement: force direction sensing may have lower sensitivity at high frequencies; the hand grip (e.g., a stylus) filters rendered forces [8]; visual cues

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Figure 1: Parallel haptic interface with 5 prismatic actuators connected to a hand-held stylus.

Figure 2: Diagram of the haptic interface.

can dominate haptic sensing [5]; and the application may not require high frequency force rendering (e.g. no hard virtual surfaces, as occurs in scientific visualization [2]).

Despite the lack of requirements, a variety of haptic interfaces have been developed, some of which are perceptually “pretty good”. Our own experience in the development of a five degree-of-freedom (DOF), in-parallel interface (see Figure 1) has indicated that force tracking bandwidths on the order of 10 Hz are sufficient for many tasks, provided this tracking is isotropic, i.e. bandwidth is at least this large in all degrees of freedom. Otherwise, the interface has perceptible “directional” characteristics, and virtual surfaces do not feel “real” due to unexpected force directions upon surface contact.

This paper describes a relatively simple modeling and control design method which has been shown to improve the bandwidth of isotropic force transmission to the user. It is well-suited to haptic interface applications because it explicitly includes the dynamics of the user’s hand (in contrast to work in robot force control), and it results in a control structure which can be easily gain scheduled (if necessary) to compensate for kinematic and dynamic changes throughout the workspace. This method uses a multivariable force control loop, and can be used
to improve the transparency of any haptic interface which incorporates force sensing.

This paper is organized as follows. The haptic interface used is described in more detail in Section 2. Section 3 outlines the performance measures and control strategy. Section 4 discusses a parameter identification technique that yields a model that can be used for inverse model control Section 5 addresses digital control implementation and presents closed-loop control results.

2 Haptic Interface Description

The University of Colorado (CU) haptic interface (Figures 1 and 2) was designed to promote high fidelity force sensing and transmission to the user in 3 DOF in translation and 2 DOF in rotation [3]. Each link consists of a thin rod controlled by a prismatic friction drive actuator. Actuators are attached to a hexagonal base via two DOF gimbals. The hand grip is a pen shaped stylus attached to three actuator rods on one end and two rods on the other. Each rod attaches to the stylus through a three axis gimbal. Force sensors are located at the tip of each rod, near the stylus, to provide feedback of measured axial rod forces to the force control loop. Accurate transmission of sensed forces to the user’s fingers is provided by light, stiff connections between the stylus and these sensors. Optical encoders embedded in the actuators measure rod displacement. A DSP computer running between 1 and 2 kHz loop rates (depending on the application) implements the force control law, in addition to performing the required kinematic transformations and data interpolation to render desired effects as the user investigates data, e.g., for scientific visualization of vector fields [2].

![Diagram](image-url)

Figure 3: Hand/Mechanism partition, actuator force $R_A$, measured rod axial force $R_I$, and rod length $L$ for a one-rod case.

![Diagram](image-url)

Figure 4: Overall control structure with separate Hand and Mechanism impedances.
2.1 Mathematical Description

There is a natural separation of the haptic interface dynamics into a component representing the interface mechanism itself, and a component representing the user’s hand. Figure 3 shows this partition for a single rod case, as well as the location of rod actuator forces $R_A$, sensed (mechanism/hand interaction) rod forces $R_I$, voluntary user forces $R_U$, and measured rod length $L$. In the multiple-rod case, the force sensors induce an unusual partition of mechanism and hand dynamics that is expedient for model identification. Here, the Mechanism impedance, $Z_M$, includes all hardware up to the force sensors, whereas the Hand impedance, $Z_H$, includes all dynamics outboard of the force sensors, i.e., the stylus, rod tip gimbals, and user’s hand lumped together.

Thinking of each of these two components as a mechanical impedance relating motions to efforts will lead to a simple model parameterizing effective mass, stiffness, and damping in that component. Although the actual models are undoubtedly more complex than this, it is shown that these models can capture the dominant multivariable dynamics and can therefore lead to improved control of rendered forces compared to separate single-input-single-output force control on each rod.

Figure 4 shows the 5-DOF force control loop resulting from the attachment of the five rods to the stylus and hand. Forces and motions are described not in Cartesian coordinates, but along the 5 moving rod axes. $G$ is the force controller.

This force loop is designed to cause the five-tuple of measured rod interaction forces, $R_I$, to track the five force loop commands (desired forces) $R_D$, using the multivariable control law $G$, which outputs five actuator forces, $R_A$. The $R_I$ are measurements of interaction forces along each rod’s long axis. Motion of the Mechanism causes changes in the distance between each actuator and the end of the rod. The 5-tuple of these rod lengths is denoted $L$. Changes in $L$ cause a reaction force from the Hand, denoted $R_H$, due to the impedance of the Hand, $Z_H$. This reaction force and the voluntary user force $R_U$ combine to form the interaction force $R_I$. In turn, the motion $L$ results from the sum of actuator forces $R_A$ and interaction forces $R_I$ acting on the Mechanism admittance $Z_M^{-1}$. The Mechanism dynamics consist primarily of mass and friction elements, which cause significant differences between the applied actuator forces $R_A$ and the forces $R_I$ applied to the Hand. Friction dominates at low frequencies, and is primarily Coulombic. At high frequencies, much of the actuator force is diverted into accelerating the Mechanism mass. Provided the force loop is stable, high loop gains over suitable bandwidths improve the tracking of desired forces $R_D$ by measured forces $R_I$, reducing the perceived friction and mass of the interface.

Although the Coulomb friction in the interface is non-linear, we employ Laplace transform representations of signals in Figure 4. The implication is that the impedance $Z_M$ relating motions and applied forces is not independent of these signals, as occurs when dynamics are strictly linear. Furthermore, $Z_M$ will vary (slowly) with location and pose in the workspace, due to kinematic nonlinearities. In other words, $Z_M$ belongs to a set of functions which is operation-dependent. Actually, $Z_H$ must also be considered a set of functions, due to the uncertainty with which any model of a physical system will have, as well as variations due to particular user dynamics, stylus grip force, and location in the workspace. Any particular functions $Z_M$ and $Z_H$ provide an
approximation to other possible members of their respective sets. Control design based on these models must therefore be robust to these variations. Theoretically, this is fraught with difficulties. Practically, this problem is often solved by including sufficient stability margins in a design based on representative members of the model set. We follow the latter approach here.

Under the condition that the user does not supply any voluntary forces (i.e., \( R_V = 0 \)), we have the following Laplace transform relations between signals in the block diagram of Figure 4, where \( Z_T (s) \) is the Total interface impedance \( Z_T (s) = Z_M (s) + Z_H (s) \):

\[
\begin{align*}
R_I (s) & = -Z_H (s) L (s) \\
R_I (s) - R_A (s) & = Z_M (s) L (s) \\
R_A (s) & = -Z_T (s) L (s) = Z_T (s) Z_H^{-1} (s) R_I (s) .
\end{align*}
\]

These definitions lead to a “plant” relation \( P (s) \) between actuator forces \( R_A \) and measured interaction forces \( R_I \):

\[
\begin{align*}
R_I (s) & = P (s) R_A (s) \\
P (s) & = Z_H (s) Z_T^{-1} (s) .
\end{align*}
\]

The decomposition into a Mechanism and Hand impedance, and the corresponding modeling and control methods discussed here, are applicable to any haptic interface, whether parallel, serial, or serial-parallel in design. Ideally, the force sensing should be located as close to the user’s fingers as possible, so that the measured forces closely represent those felt by the user, and so that the force loop causes these to track the desired forces.

3  Performance Measures and Control Strategy

3.1 Performance Measures

The goal of the force controller is to increase the bandwidth of isotropic force transmission. Here, we investigate multivariable (MIMO) control, since independent SISO control on each rod is extremely limited in its ability to affect isotropy. The force transmission transfer function \( T (j \omega) \) relating desired forces \( R_D \) to measured interaction forces \( R_I \) can be derived from the block diagram in Figure 4 and equation (2):

\[
\begin{align*}
R_I (j \omega) & = T (j \omega) R_D (j \omega) \\
T (j \omega) & = (I + P (j \omega) G (j \omega)) P (j \omega) G (j \omega) .
\end{align*}
\]

Ideally, \( T (j \omega) = I \), so that the user feels the desired force \( R_D \) in all degrees of freedom and at all frequencies. This can sometimes be achieved (typically only at low frequencies) by a minimum singular value \( \sigma (P (j \omega) G (j \omega)) \) which is large compared to 1. At higher frequencies, \( \sigma (P (j \omega) G (j \omega)) \) necessarily becomes smaller to ensure stability. In fact it is the maximum singular value \( \sigma (P (j \omega) G (j \omega)) \) which affects stability. When the condition number of \( P (j \omega) G (j \omega) \) is large, the minimum and maximum singular values are widely separated, resulting in a wide
variation in loop gain in different directions. A maximum singular value near 1 (for stability) may then imply a very small value of the minimum singular value at that same frequency, resulting in poor force tracking in some direction. As a result, the rendered force vector may be corrupted in magnitude and/or direction. Therefore, to obtain isotropic force tracking, we seek to reduce the condition number of $P(j\omega)G(j\omega)$, particularly at frequencies where $\sigma(P(j\omega)G(j\omega))$ is near 1. This extends the bandwidth of isotropic force tracking, and simultaneously promotes better stability margins. This criterion is not sufficient, though; even if the condition number of $T$ is 1 (a unitary matrix), $T$ may still cause vector rotation if the eigenvalues of $T$ are complex. Hence we will want to specify that $\sigma(T(j\omega) - I)$ is suitably small, since this bounds the magnitude of the error between a desired force vector $R_D$ and its rendered value $R_I$ at each frequency. To be specific, we define the bandwidth of isotropic force tracking as the maximum frequency where $\sigma(T(j\omega) - I)$ is less than $1 - 1/\sqrt{2} \approx 0.293$ (analogous to the 3 dB bandwidth definition for scalar systems). Figure 5 shows the singular values of $(T(j\omega) - I)$ under independent

![Figure 5: Singular values of $(T(j\omega) - I)$ under SISO control.](image)

SISO control. Note that the vertical axis is magnitude (not in dB). The isotropic force transmission bandwidth (as defined above) is less than 1 Hz, with wide directional variation at low frequencies. Multivariable control will be investigated, relative to the SISO baseline, to improve isotropic bandwidth by “compressing” singular values together, reducing the condition number.

Stability of the force loop can be assessed using the multivariable Nyquist criterion [12]. This multivariable version of the Bode and Nyquist analyses uses the eigenvalues of the loop transfer function matrix, counting encirclements of the critical point in a similar way as for scalar systems. Gain and phase margins can be similarly inferred, provided that the eigenvectors are nearly orthogonal.

It is important to mention that a stable force loop may not be sufficient when the system contains an outer impedance loop, since the overall system may be destabilized by this outer loop. Figure 6 shows a typical situation where the desired force $R_D$ is a function of haptic interface motion $L$. Here the interface is attempting to render a desired impedance $Z_D$, and $Z_{CL}(j\omega) = Z_H^{-1}(j\omega) T(j\omega)$. Note that in the ideal case where $T(j\omega) = I$,  

<table>
<thead>
<tr>
<th>Frequency, Hz</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^-1</td>
<td>0</td>
</tr>
<tr>
<td>10^-1</td>
<td>0.5</td>
</tr>
<tr>
<td>10^-1</td>
<td>1.0</td>
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<td>1.5</td>
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<tr>
<td>10^-1</td>
<td>2.0</td>
</tr>
<tr>
<td>10^-1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

6
\( R_I = R_D = Z_D(L_D - L) \), and the user feels only \( Z_D \). That is, the Mechanism impedance \( Z_M \) is suppressed by the force control loop, resulting in a perfectly transparent interface. Short of this ideal, improvements in isotropic force tracking increase the fidelity of the rendered \( Z_D \), provided the outer loop is stable, with adequate margins. Stability of this loop can be assessed using the multivariable Nyquist criterion. Other criteria, e.g. passivity [15] could also be applied.

![Figure 6: Outer impedance loop](image)

### 3.2 Control Strategy

Given the objective of “compressing” the singular values of the loop gain, an inverse model control strategy is pursued. If \( \hat{Z}_H(s) \) and \( \hat{Z}_T(s) \) are models of the Hand and Total impedances respectively, then a MIMO controller can take the form:

\[
R_A(s) = G(s) R_E(s)
\]

with

\[
G(s) = g(s) \hat{Z}_T(s) \hat{Z}_H^{-1}(s) = g(s) \hat{P}^{-1}(s)
\]  

(3)

where \( \hat{P}^{-1}(s) \) is the (inverse) plant model and \( g(s) \) is a simple scalar compensator used to further shape the frequency response by increasing low-frequency gain and providing roll-off at higher frequencies. To the extent that the plant model matches the plant, the singular value magnitudes of \( P(s) \hat{P}^{-1}(s) \) will be compressed into a narrow band around 0 dB, allowing force tracking to be similarly effective in all directions. The second order polynomial forms used for the impedances in \( \hat{P} \) (see below) are well-suited to model inversion, since the resulting model has relative degree zero, and the inverse is realizable without large high frequency gains.

### 4 Modeling

The goal of this section is to obtain a parametric model of the plant transfer function of the form:

\[
\hat{P}(s) = \hat{Z}_H(s) \hat{Z}_T^{-1}(s).
\]

(4)

To this end, we first seek separate Mechanism and Hand impedance models. Previous work [6] suggests that a second-order linear model of the Mechanism provides considerable accuracy in modeling the non-Coulomb friction portion of the Mechanism dynamics at low frequencies. The model is of the form:

\[
\hat{Z}_M(s) = \hat{M}_M s^2 + \hat{B}_M s + \hat{K}_M.
\]
In this paper we will examine the feasibility and performance of Hand and Total impedance models of the same form:

\[
\begin{align*}
\hat{Z}_H (s) &= \hat{M}_H s^2 + \hat{B}_H s + \hat{K}_H \\
\hat{Z}_T (s) &= \hat{M}_T s^2 + \hat{B}_T s + \hat{K}_T.
\end{align*}
\]

(5)

The mass matrices \(\hat{M}_H\) and \(\hat{M}_T\), damping matrices \(\hat{B}_H\) and \(\hat{B}_T\), and stiffness matrices \(\hat{K}_H\) and \(\hat{K}_T\) will be obtained using a least squares fitting of frequency domain data in Section 4.2 using equation (1).

### 4.1 Hand and Total Impedance Frequency Response Data

The method for obtaining the frequency response data is similar for the Hand and Total impedances. For brevity, we discuss only the Hand. The relationship at a frequency \(\omega\) between measured rod forces \(R_i\) and measured rod displacements \(L\) in the absence of voluntary user force \(R_V\) is the Hand impedance at that frequency:

\[
R_i (j\omega) = -Z_H (j\omega) L (j\omega).
\]

Let \(Z_H (j\omega) = [z_{ij} (j\omega)]\). To obtain the scalar frequency responses of the \(z_{ij}\) at a set of frequencies \([\omega_1, \omega_2, ..., \omega_N]\), the system must be excited at those frequencies. Let \(R_A (t)\) denote the 5-tuple of actuator input forces at time \(t\), and consider the following five sets of actuator inputs over a time period \(\tau\), to be used in a set of five data collection experiments:

\[
\begin{align*}
R_{A_1} (t) &= r_f (t) \begin{bmatrix} -1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \\
R_{A_2} (t) &= r_f (t) \begin{bmatrix} 1 & -1 & 1 & 1 & 1 \end{bmatrix}^T \\
R_{A_3} (t) &= r_f (t) \begin{bmatrix} 1 & 1 & -1 & 1 & 1 \end{bmatrix}^T \\
R_{A_4} (t) &= r_f (t) \begin{bmatrix} 1 & 1 & 1 & -1 & 1 \end{bmatrix}^T \\
R_{A_5} (t) &= r_f (t) \begin{bmatrix} 1 & 1 & 1 & 1 & -1 \end{bmatrix}^T
\end{align*}
\]

(6)

where \(r_f (t)\) is a pseudo-random signal

\[
r_f (t) = \sum_{k=1}^{N} A \cos (\omega_k t + \alpha_k)
\]

and the \(\alpha_k\) are uniformly distributed random phases in the interval \([0, 2\pi]\), and \(A\) is selected to produce an adequate signal to noise ratio in the measured data. If \(N\) is the number of time samples during time \(\tau\), i.e., \(\tau = N t_s\) with \(t_s\) being the sample period, then the frequencies \(\omega_k\) must be integer multiples of \(\frac{2\pi}{t_s}\) to avoid errors in the Fast Fourier Transform (FFT). For each experiment (for \(\ell = 1, \ldots, 5\)), by taking the FFT of the measured rod forces

\(^1\)the subscript index \(j\) should not be confused with the imaginary \(j\).
$R_{Ii}^m(j\omega)$ and rod lengths $L_{I}^m(j\omega)$, $m = 2, \ldots, 6$, one obtains, at each frequency $^2 \omega$:

$$
R_{Ii}(j\omega) = \begin{bmatrix}
R_{I1}^2(j\omega) \\
R_{I2}^3(j\omega) \\
R_{I3}^4(j\omega) \\
R_{I4}^5(j\omega) \\
R_{I5}^6(j\omega)
\end{bmatrix} = -Z_H(j\omega) \begin{bmatrix}
L_{I1}^2(j\omega) \\
L_{I2}^3(j\omega) \\
L_{I3}^4(j\omega) \\
L_{I4}^5(j\omega) \\
L_{I5}^6(j\omega)
\end{bmatrix}
\Rightarrow R_{Ii}(j\omega) = -Z_H(j\omega)L_{I}(j\omega).
$$

By concatenating these results by column into matrix form, one obtains

$$
\mathcal{R}_I(j\omega) = -Z_H(j\omega)\mathcal{L}(j\omega)
$$

(7)

where

$$
\mathcal{R}_I(j\omega) = \begin{bmatrix}
R_{I1}(j\omega) & R_{I2}(j\omega) & \cdots & R_{I5}(j\omega)
\end{bmatrix} \\
\mathcal{L}(j\omega) = \begin{bmatrix}
L_1(j\omega) & L_2(j\omega) & \cdots & L_5(j\omega)
\end{bmatrix}
$$

are both $5 \times 5$ matrices at each frequency $\omega$. Hence, the empirical (measured) impedance $Z_H(j\omega)$ can be found as

$$
Z_H(j\omega) = -\mathcal{R}_I(j\omega)\mathcal{L}(j\omega)^{-1}
$$

provided the columns of $\mathcal{L}$ are linearly independent. In our experiments, the choice (6) provided sufficiently independent columns in $\mathcal{L}$. Other choices are possible, but the particular choice of $R_{Ae}(t)$ affects the quality of the frequency response: if one of the rods is not sufficiently actuated, data will be corrupted by quantization noise, friction, etc. This is most likely to occur at high frequencies when the amplitude of motion is reduced and friction may dominate. We observed this deficiency, for example, when only one rod was excited at a time. This caused so little motion in some rods that the empirical impedance was highly inaccurate.

The model for $Z_T$ can be constructed in a similar manner:

$$
Z_T(j\omega) = -\mathcal{R}_A(j\omega)\mathcal{L}(j\omega)^{-1}
$$

where $\mathcal{R}_A(j\omega) = \begin{bmatrix}
R_{A1}(j\omega) & R_{A2}(j\omega) & \cdots & R_{A5}(j\omega)
\end{bmatrix}$ is the measured actuator force vector. Although not directly needed, the Mechanism impedance itself can also be found from

$$
Z_M(j\omega) = (\mathcal{R}_I(j\omega) - \mathcal{R}_A(j\omega))\mathcal{L}(j\omega)^{-1}.
$$

These frequency domain matrices are now parameterized to extract mass, spring, and damping matrix descriptions of the Hand and Total impedances.

---

$^2$The superscript in $R_{Ii}^m$, $m = 2, \ldots, 6$ is used to index rod number and is not a power exponent.
4.2 $M, B, K$ Model of $Z_H$ and $Z_T$

The identification of the mass, spring, and damping matrices for the Hand and Total impedances each follow the same method. We discuss the method only for the Hand, but will show results for both the Hand and Total impedances.

For each element $z_{ij} (j\omega)$ of the Hand impedance $Z_H$, we wish to find a model of the form

$$
\hat{z}_{ij} (j\omega) = \begin{bmatrix} (j\omega)^2 & j\omega & 1 \\
\hat{m}_{ij} \\
\hat{b}_{ij} \\
\hat{k}_{ij} \end{bmatrix}.
$$

Stacking these equations by frequency, we get:

$$
\hat{Z}_{ij} = \begin{bmatrix}
\hat{z}_{ij} (\omega_1) \\
\hat{z}_{ij} (\omega_2) \\
\vdots \\
\hat{z}_{ij} (\omega_N)
\end{bmatrix} = \begin{bmatrix}
(j\omega_1)^2 & j\omega_1 & 1 \\
(j\omega_2)^2 & j\omega_2 & 1 \\
\vdots & \vdots & \vdots \\
(j\omega_N)^2 & j\omega_N & 1
\end{bmatrix} \begin{bmatrix}
\hat{m}_{ij} \\
\hat{b}_{ij} \\
\hat{k}_{ij}
\end{bmatrix},
$$

which we can rewrite as

$$
\hat{Z}_{ij} = \phi^H \theta_{ij},
$$

where “$H$” denotes the conjugate transpose. The real parameter 3-tuple $\theta_{ij} = [\hat{m}_{ij}, \hat{b}_{ij}, \hat{k}_{ij}]^T$ can be found by solving the least squares problem:

$$
\hat{\theta}_{ij} = \arg \min \left( Z_{ij} - \phi^H \theta_{ij} \right)^H \left( Z_{ij} - \phi^H \theta_{ij} \right)
$$

where $Z_{ij}$ is the $N$-tuple of measured impedances $z_{ij} (j\omega)$. Although the magnitude of the frequency response of each $Z_{ij}$ is relatively constant between 0 and 7 Hz, it then follows an upward trend at a rate of roughly 40 dB/decade (see Figure 7). The plain least squares minimization thus weights errors at the higher frequencies heavily, since the magnitude is larger at the higher frequencies. This causes greater modeling error at low frequencies. In order for the model to be equally accurate at all frequencies of identification, we weight the measured data and the information matrix $\phi$ with a simple model of the inverse of the frequency response of the data

$$
\hat{h} (j\omega) = \frac{1}{(j\omega + 7 \cdot 2\pi)^2}.
$$
The weighted data is

$$
\tilde{Z}_{ij} = \begin{bmatrix}
  h(\omega_1) z_{ij}(\omega_1) \\
  h(\omega_2) z_{ij}(\omega_2) \\
  \vdots \\
  h(\omega_N) z_{ij}(\omega_N)
\end{bmatrix},
$$

$$
\tilde{\phi}^H = \begin{bmatrix}
  h(\omega_1) (j\omega_1)^2 & h(\omega_1) j\omega_1 & h(\omega_1) 1 \\
  h(\omega_2) (j\omega_2)^2 & h(\omega_2) j\omega_2 & h(\omega_2) 1 \\
  \vdots & \vdots & \vdots \\
  h(\omega_N) (j\omega_N)^2 & h(\omega_N) j\omega_N & h(\omega_N) 1
\end{bmatrix}.
$$

The modified minimization problem can be written as

$$
\hat{\theta}_{ij} = \arg \min \left( \tilde{Z}_{ij} - \tilde{\phi}^H \hat{\theta}_{ij} \right)^H \left( \tilde{Z}_{ij} - \tilde{\phi}^H \hat{\theta}_{ij} \right).
$$

The least squares solution to this modified problem is (see e.g. [10])

$$
\hat{\theta}_{ij} = \left( \text{Real} \left( \tilde{\phi} \tilde{\phi}^H \right) \right)^{-1} \text{Real} \left( \tilde{\phi} \tilde{Z}_{ij} \right).
$$

Parameter 3-vectors $\hat{\theta}_{ij}$ are found in this way for each of the 25 elements of the $5 \times 5$ ZM matrix. To obtain a model for the total impedance $Z_T$, this process is repeated, with $Z_{ij}$ derived from the elements of the empirical $ZT$ data matrix at each frequency. Although there are a large number of parameters, the process is easily automated and involves only a $5 \times 5$ matrix inverse at each stage.

### 4.3 Modeling results

$Z_H$ and $Z_T$ were modeled using the method above with excitation at the 28 frequencies 1, 2, 3, ..., 9, 10, 15, 20, ..., 100 Hz. The singular values of $Z_H$ and its model $\tilde{Z}_H$ are shown in Figure 7. Those for $Z_T$ and its resulting model $\tilde{Z}_T$ are shown in Figure 8. It can be seen from these plots that the impedances are not those of strict second-order matrix polynomials. Nevertheless, this relatively simple approximation is accurate enough to capture salient system dynamics. To see this we look at singular values of $P(j\omega) \cdot \tilde{\phi}^{-1}(j\omega)$, which ideally would all be 1. Figure 9 shows that these singular values occupy a magnitude band within ±10 dB for frequencies up to 30 Hz, and less than +5/-7 dB for frequencies between 2 and 20 Hz. The plant alone had a singular value span ranging from 25 to 50 dB for frequencies up to 30 Hz, hence this model inversion component of the controller has has reduced anisotropy by a factor ranging from 5 dB to 38 dB over this frequency range.

### 5 Control Implementation

The controller takes the form:

$$
G(s) = g(s) \tilde{Z}_T(s) \tilde{Z}_H^{-1}(s).
$$
Figure 7: Singular values of $Z_H$ (solid) and $\tilde{Z}_H$ (dashed).

Figure 8: Singular values of $Z_T$ (solid) and $\tilde{Z}_T$ (dashed).

Figure 9: Singular values of $P \cdot \tilde{P}^{-1}$. 
Before addressing the scalar compensator design \( g(s) \), we first discuss two implementation issues related to the inverse model component \( \hat{P}^{-1}(s) \). First, model right-half-plane poles and zeros may appear from the least squares parameter matching; second, the continuous multivariable inverse model must be discretized for implementation.

### 5.1 Right Half Plane Poles and Zeros in Model

Since the Hand and Total impedances are real physical systems, their impedance realization \( \hat{Z}_H(s) \) and \( \hat{Z}_T(s) \) should have zeros in the open complex left half plane (OLHP). Unfortunately, due to noise in the data measurements, the least squares algorithm may return impedance models with zeros in the closed complex right half plane (CRHP). CRHP zeros of \( \hat{Z}_H(s) \) cause \( G(s) \) to be unstable. This is undesirable since it makes the controller implementation difficult to debug: a frequency response of the controller cannot be obtained before the loop is closed. CRHP roots of \( \hat{Z}_T(s) \) cause \( G(s) \) to be non-minimum phase and are also undesirable since they induce phase loss with an increase in magnitude, limiting closed-loop bandwidth [14]. These two issues can often be remedied by “positizing” the mass, spring, and damping matrices of \( \hat{Z}_H(s) \) and \( \hat{Z}_T(s) \). If \( VDV^{-1} \) is the eigenvector decomposition of a matrix \( M \), then we define \( M_P \) to be the positized version of \( M \) as follows:

\[
M_P = V|D|V^{-1}
\]

where \(|\cdot|\) is the absolute value of matrix elements. Note the eigenvectors and eigenvalue magnitudes of the parameter matrices are left unchanged under positizing.

Once the terms of the impedance model have been positized, the roots of the impedance models \( \hat{Z}_H(s) \) and \( \hat{Z}_T(s) \) are checked for the presence of remaining CRHP roots. Positizing normally is sufficient, but if CRHP roots are found, one can return to the original matrices, symmetrize them, and then positize them. This will result in positive definite mass, spring, and damping matrices, ensuring OLHP roots [9]. Unfortunately, we found that symmetrization can significantly reduce the accuracy of the model \( \hat{P} \) by altering the eigenvectors and the magnitudes of the eigenvalues, and should be avoided if possible.

### 5.2 Discrete-Time Implementation

To implement the controller in equation (3), define

\[
y(s) = \hat{Z}_T^{-1}(s) R_A(s) = \hat{Z}_H^{-1}(s) g(s) R_E(s). \]

Then, using the plant models from equations (5),

\[
g(s) R_E(s) = \hat{M}_H s^2 y(s) + \hat{B}_H s y(s) + \hat{K}_H y(s)
\]

\[
\Rightarrow s^2 y(s) = g(s) \hat{M}_H^{-1} R_E(s) - \hat{M}_H^{-1} \hat{B}_H s y(s) - \hat{M}_H^{-1} \hat{K}_H y(s)
\]

and

\[
R_A(s) = \hat{M}_T s^2 y(s) + \hat{B}_T s y(s) + \hat{K}_T y(s).
\]
Figure 10: Continuous implementation of the plant inverse model.

Figure 10 shows the continuous-time version of the controller. A simple way to find a “discrete equivalent” is to replace the integrators $\frac{1}{s}$ with $\frac{z}{z-1}$, as shown in Figure 11. One could also use $\frac{z^2}{z-1}$, but this would induce extra delay, reducing the stability margin. To avoid the algebraic loop in Figure 11, the transfer function between $Y_1$

and $R_A$ can be calculated:

$$Y_5(k) = Y_5(k-1) + t_s Y_4(k)$$

$$\Rightarrow Y_5(k) - t_s Y_4(k) = Y_5(k-1)$$

$$Y_4(k) = Y_4(k-1) + t_s \hat{M}_H^{-1} \left( Y_1(k) - \hat{B}_H Y_4(k) - \hat{K}_H Y_5(k) \right)$$

$$\Rightarrow Y_4(k) + t_s \hat{M}_H^{-1} \hat{B}_H Y_4(k) + t_s \hat{M}_H^{-1} \hat{K}_H Y_5(k) = t_s \hat{M}_H^{-1} Y_1(k) + Y_4(k-1).$$
It is expedient to place these equations into matrix form:

\[
\begin{bmatrix}
Y_5 (k - 1) \\
t_s \hat{M}^{-1}_H Y_1 (k) + Y_4 (k - 1)
\end{bmatrix}
= \begin{bmatrix}
I & -t_s I \\
t_s \hat{M}^{-1}_H \hat{K}_T & I + t_s \hat{M}^{-1}_H \hat{B}_H
\end{bmatrix}
\begin{bmatrix}
Y_5 (k) \\
Y_4 (k)
\end{bmatrix}.
\]

Defining the (constant) $10 \times 10$ matrix $A$ as

\[
A = \begin{bmatrix}
I & -t_s I \\
t_s \hat{M}^{-1}_H \hat{K}_T & I + t_s \hat{M}^{-1}_H \hat{B}_H
\end{bmatrix}^{-1}
\]

allows us to write

\[
\begin{bmatrix}
Y_5 (k) \\
Y_4 (k)
\end{bmatrix} = A \begin{bmatrix}
Y_5 (k - 1) \\
Y_4 (k - 1)
\end{bmatrix} + A \begin{bmatrix}
0 \\
t_s \hat{M}^{-1}_H
\end{bmatrix} Y_1 (k).
\]

From Figure 11, the actuator force is

\[
R_A (k) = \hat{M}_T Y_3 (k) + \hat{B}_T Y_4 (k) + \hat{K}_T Y_5 (k)
\]

where $Y_3 (k)$ is

\[
Y_3 (k) = \frac{Y_4 (k) - Y_4 (k - 1)}{t_s}.
\]

The design and discretization of $g (s)$ is discussed in Section 5.3.

Note that any model found with the method above has the same structure. Thus if hand or mechanism dynamics vary sufficiently throughout the workspace, it may be necessary to develop a suite of models and corresponding controllers, and gain schedule these as a function of position in the workspace. Having identical structure, the suite of controllers can be gain scheduled by simple parameter interpolation of the $A$, $\hat{M}^{-1}_H$, $\hat{M}_T$, $\hat{B}_T$, and $\hat{K}_T$ matrices. In our case, a single controller using values at the center of the workspace has yielded good results, in all but the edges of the workspace.

### 5.3 Scalar Compensator Design

Once the goal of singular value “compressing” is achieved with the inverse model controller, it remains to shape the open-loop frequency response to provide high gain at low frequencies to improve force tracking, and roll-off at the higher frequencies to subdue unmodeled dynamics. In our work, the scalar $g (s)$ factor was chosen to include a gain, 3 notch filters to address specific peaks in the open-loop response, a lag filter to boost low frequency gain, and 2 first-order low-pass filters to roll off high frequency gain, as follows:

\[
g (s) = k \left( \frac{s^2 + 2 \zeta_1 \omega_1 s + \omega_1^2}{s^2 + 2 \zeta_2 \omega_2 s + \omega_2^2} \right) \left( \frac{s^2 + 2 \zeta_3 \omega_3 s + \omega_3^2}{s^2 + 2 \zeta_4 \omega_4 s + \omega_4^2} \right) \left( \frac{s + z_1}{s + p_1} \right) \left( \frac{p_2}{s + p_2} \right) \left( \frac{p_3}{s + p_3} \right)
\]

where

\[
k = 0.85
\]

\[
\omega_1 = 100 \cdot 2 \pi \text{ rad/sec}
\]

\[
\omega_2 = 55 \cdot 2 \pi \text{ rad/sec}
\]
\[ \omega_3 = 17 \cdot 2\pi \text{ rad/sec} \\
\zeta_1 = 0.3 \\
\zeta_2 = 1.0 \\
z_1 = 30 \cdot 2\pi \text{ rad/sec} \\
p_1 = 1 \cdot 2\pi \text{ rad/sec} \\
p_2 = 100 \cdot 2\pi \text{ rad/sec} \\
p_3 = 200 \cdot 2\pi \text{ rad/sec} . \]

This continuous scalar compensator is discretized using \( z = e^{s T} \) and matching DC gains (see e.g. [11]).

### 5.4 Measured Results

Figure 12 shows the Nyquist and Bode plots for the open-loop frequency response, measured as the relation between force errors to measured forces in the hardware implementation. The achieved gain margin is 9 dB, and the phase margin is about 20 degrees.

![Nyquist and Bode plots](image)

**Figure 12:** Eigenvalues of measured \( P(s) G(s) \).

Figure 13 shows the singular values of \( (T(j\omega) - I) \) for the MIMO controller. These singular values are ideally 0. The isotropic bandwidth for the MIMO controller is 7 Hz. It can be seen that the singular values of \( (T(j\omega) - I) \) are much smaller for the MIMO control case than for the independent SISO control case (see Figure 5) for frequencies up to 10 Hz. The large singular value of \( (T(j\omega) - I) \) at 15 Hz reflects a peak in \( T((j\omega)) \) of about 6 dB, which results from the relatively small phase margin.

The MIMO results are a very large improvement over independent SISO design results. The “feel” of the interface under the new control scheme is noticeably sharper: free space feels freer, and virtual walls are stiffer with less directional distortion. The interface does have a slight “alive” feel to it due to the under damped closed loop mode at 15 Hz. If desired, the control design could be iterated by changing the weighting function \( h \) in the least squares model fitting procedure, or by changing the scalar controller \( g \) to achieve better stability margins. It is also
possible that the nonlinear Coulomb friction present in the Mechanism may be biasing the estimates of impedance parameters, particularly damping, and it may be necessary to take FFT data under different excitation levels to obtain further performance improvements.

6 Conclusion

Transparency of haptic interfaces can be improved by force control loops, which cause rendered forces to more accurately track desired forces. Isotropic tracking is important in haptic interfaces, so that rendered objects produce reaction forces not only with appropriate magnitudes, but also in appropriate directions. The modeling and control methods presented here were shown to lead to effective isotropic force control in a particular parallel haptic interface. These methods are applicable to all haptic interfaces equipped with force sensing.

Simplicity of the models derives from the second-order polynomial matrix models for constituent impedances. These capture the dynamics of haptic interface and human hand with sufficient accuracy to enable significant improvement in isotropic bandwidth over independent SISO control designs. The multivariable controller greatly improves the feel of rendered objects on the CU haptic interface.

Force control may be sufficient for haptic interfaces which directly render data values as forces (as in some applications of scientific visualization). Other applications may render forces which are a function of hand position, which results in an “impedance” loop enclosing the inner force loop. Here, the force loop can provide an improvement in rendered impedance accuracy.

The structure of the controller lends itself to gain scheduling, when dynamics of the hand or interface are strong functions of position or pose. This is because all controllers generated from linear models at various locations have the same structure, allowing a straightforward interpolation of control parameters as a function of location.
References


